XVII International Zhautykov Olympiad in Mathematics
Almaty, 2021, january 9, second day
(Time allowed is 4.5 hours. Each problem is worth 7 points)

№4. A circle with radius r is inscribed in the triangle ABC. Circles with radii r_1, r_2, r_3 ($r_1, r_2, r_3 < r$) are inscribed in the angles A, B, C so that each touches the incircle externally. Prove that $r_1 + r_2 + r_3 \geq r$.

№5. On a party with 99 guests, hosts Ann and Bob play a game (the hosts are not regarded as guests). There are 99 chairs arranged in a circle; initially, all guests hang around those chairs. The hosts take turns alternately. By a turn, a host orders any standing guest to sit on an unoccupied chair c. If some chair adjacent to c is already occupied, the same host orders one guest on such chair to stand up (if both chairs adjacent to c are occupied, the host chooses exactly one of them). All orders are carried out immediately. Ann makes the first move; her goal is to fulfill, after some move of hers, that at least k chairs are occupied. Determine the largest k for which Ann can reach the goal, regardless of Bob's play.

№6. Let $P(x)$ be a nonconstant polynomial of degree n with rational coefficients which can not be presented as a product of two nonconstant polynomials with rational coefficients. Prove that the number of polynomials $Q(x)$ of degree less than n with rational coefficients such that $P(x)$ divides $P(Q(x))$

a) is finite;

b) does not exceed n.