XVII International Zhautykov Olympiad in Mathematics
Almaty, 2021, january 8, first day
(Time allowed is 4.5 hours. Each problem is worth 7 points)

№1. Prove that for some positive integer n the remainder of 3^n when divided by 2^n is greater than 10^{2021}.

№2. In a convex cyclic hexagon $ABCDEF$ $BC = EF$ and $CD = AF$. Diagonals AC and BF intersect at point Q, and diagonals EC and DF intersect at point P. Points R and S are marked on the segments DF and BF respectively so that $FR = PD$ and $BQ = FS$. **The segments** RQ and PS intersect at point T. Prove that the line TC bisects the diagonal DB.

№3. Let $n \geq 2$ be an integer. Elwyn is given an $n \times n$ table filled with real numbers (each cell of the table contains exactly one number). We define a *rook set* as a set of n cells of the table situated in n distinct rows as well as in n distinct columns. Assume that, for every rook set, the sum of n numbers in the cells forming the set is nonnegative.

By a move, Elwyn chooses a row, a column, and a real number a, and then he adds a to each number in the chosen row, and subtracts a from each number in the chosen column (thus, the number at the intersection of the chosen row and column does not change). Prove that Elwyn can perform a sequence of moves so that all numbers in the table become nonnegative.