
Scheme for №1.

1. It has been noticed that for large n, if the integer part of the quotient of 3^n and 2^n is odd, then $n+1$ is a solution: .... **2 points.**
2. Minor mistakes: .... **-1 point.**

Scheme for №2.

Marking scheme for first solution.

General remarks:

a) Full solution: .... **7 points.**
b) For incomplete computations (coordinates, complex numbers, vectors, trigonometry, etc.): . **0 points.**

Partial achievements:

1. Proving that $CPFQ$ is a parallelogram: .... **0 points.**
2. Proving that $CQRD$ and (or) $CPSB$ are parallelograms: .... **1 point.**
3. Proving that $BX = DY$ and (or) $DX = BY$: .... **4 points.**
4. Proving that AD and BE are common internal tangents lines of ω_1 and (or) ω_2: .... **1 point.**
5. Proving that $X \in \omega_1$ and (or) $Y \in \omega_2$: .... **1 point.**

Paragraphs 4 and 3 are not additive.

Paragraphs 5 and 3 are not additive.

Marking scheme for second solution.

General remarks:

a) Full solution: .... **7 points.**
b) For incomplete computations (coordinates, complex numbers, vectors, trigonometry, etc.): . **0 points.**

Partial achievements:

1. Proving that $CPFQ$ is a parallelogram: .... **0 points.**
2. Proving that $CQRD$ and (or) $CPSB$ are parallelograms: .... **1 point.**
3. Proving that the points C, M and T are collinear, where $FRMS$ is a parallelogram: .... **4 points.**

Scheme for №3.

No points are provided for the proofs of Propositions 1 and 2

No points are deducted ir Proposition 1 is used without a proof.

If Proposition 2 is used without a proof .... **1 point is deducted**

In the sequel, we list some advantages to the official solutions. The points within one solution are additive; the points from different solutions are not additive.

Towards Solution 1

No points are provided for just a proof of the Lemma.

(1.1) If the Lemma is used without a proof, **no points are deducted**.

(1.2) Formulation of the Claim, **and** reducing the problem statement to the Claim .... **1 point**

Caveat. The proof of the following statement (following from Proposition 2) is regarded as satisfying (1.2): *It suffices to prove that there is a table filled with nonnegative numbers whose rook set sums are respectively equal to those in Elwyn's table.*

(1.3) Proof of the Claim .... **6 points**

No points are provided for just showing that one can decrease several numbers in a good table so that the table remains good, and all its cells become blocked.

Towards Solution 2

(2.1) Only a proof of Step 1 .... **5 points**

(2.2) Only a proof of Step 2 .... **1 point**

(2.3) Putting all together .... **1 point**

Caveat. The point for (2.3) is provided only if the parts (2.1) and (2.2) are essentially complete, possibly with minor flaws.

Towards Solution 3

(3.1) Introducing two convex cones G and T, and showing that both are closed .... **1 point**

(3.2) Explicit description of all supporting linear functions of T .... **3 points**

(3.3) Finishing the proof using the mentioned description .... **3 points**

Scheme for №4.

Marking scheme for first solution.

General remarks:

Full solution: .... **7 points.**

Partial achievements:

1. Considering of the circles of radii r_1', r_2', r_3' (with clear description of their obtaining): ...... **3 points.**
2. Stating (without proof), that $r_1 \geq r_1'$: .... **1 points.**
3. Proving that $r_1 \geq r_1'$: .... **2 points.**
4. Stating (without proof), that $r_1' + r_2' + r_3' = r$: .... **1 point.**
5. Proving that $r_1' + r_2' + r_3' = r$: .... **2 points.**

Paragraphs 2 and 3 are not additive.

Paragraphs 4 and 5 are not additive.

Marking scheme for second solution.

General remarks:

Full solution: .... **7 points.**

Partial achievements:

1. Proving one of the equalities

- $\sin\frac{\angle A}{2} = \frac{r - r_1}{r + r_1}$ or
- $r_1 = r \cdot \dfrac{1 - \sin\frac{\angle A}{2}}{1 + \sin\frac{\angle A}{2}}$ or
- $r_1 = r \cdot \mathrm{tg}^2\left(\frac{\pi}{4} - \frac{\angle A}{4}\right)$:

.... **3 points.**

2. Stating that $\sin\frac{\angle A}{2} + \sin\frac{\angle B}{2} + \sin\frac{\angle C}{2} \leq 3 \cdot \sin\dfrac{\frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{2}}{3}$: .... **2 points.**

Nevertheless stating (without proof) that $\sin\frac{\angle A}{2} + \sin\frac{\angle B}{2} + \sin\frac{\angle C}{2} \leq \frac{3}{2}$: .... **1 point.**

3. Formulation (with reffering to Jensen's inequality) one of the following inequalities:

- $\dfrac{1-\sin\frac{\angle A}{2}}{1+\sin\frac{\angle A}{2}}+\dfrac{1-\sin\frac{\angle B}{2}}{1+\sin\frac{\angle B}{2}}+\dfrac{1-\sin\frac{\angle C}{2}}{1+\sin\frac{\angle C}{2}}\geq 3\cdot\dfrac{1-\sin\dfrac{\frac{\angle A}{2}+\frac{\angle B}{2}+\frac{\angle C}{2}}{3}}{1+\sin\dfrac{\frac{\angle A}{2}+\frac{\angle B}{2}+\frac{\angle C}{2}}{3}}$ or

- $\operatorname{tg}^2\left(\frac{\pi}{4}-\frac{\angle A}{4}\right)+\operatorname{tg}^2\left(\frac{\pi}{4}-\frac{\angle B}{4}\right)+\operatorname{tg}^2\left(\frac{\pi}{4}-\frac{\angle C}{4}\right)\geq 3\cdot\operatorname{tg}^2\dfrac{3\cdot\frac{\pi}{4}-\frac{\angle A}{4}-\frac{\angle B}{4}-\frac{\angle C}{4}}{3}$:

.... **2 points.**

4. Proving any of the inequalities

- $\dfrac{1-\sin\frac{\angle A}{2}}{1+\sin\frac{\angle A}{2}}+\dfrac{1-\sin\frac{\angle B}{2}}{1+\sin\frac{\angle B}{2}}+\dfrac{1-\sin\frac{\angle C}{2}}{1+\sin\frac{\angle C}{2}}\geq 1$ or

- $\operatorname{tg}^2\left(\frac{\pi}{4}-\frac{\angle A}{4}\right)+\operatorname{tg}^2\left(\frac{\pi}{4}-\frac{\angle B}{4}\right)+\operatorname{tg}^2\left(\frac{\pi}{4}-\frac{\angle C}{4}\right)\geq 1$:

.... **4 points.**

Only paragraphs 1 and 2, 1 and 3, 1 and 4 can be additive.

Scheme for №5.

Caveat. In what follows, by a *strategy* we always mean an explicit description of player's actions. Phrases like "we act so that ⟨*some property*⟩ is preserved" usually need some explanations, and they will be undergraded if such explanation is missing.

(O) **No points** are given for the following initial steps:

(O1) Just claiming the answer;

(O2) Noticing that F does not decrease;

(O3) Just introdicing a notion of a stable situation.

(A) Presenting a strategy ensuring that Ann reaches $F = 34$, with a correct proof that it works **4 points**

(A1) Only presenting a correct strategy allowing Ann to reach $F = 34$, without a correct proof **2 points**

(A2) Only presenting, with a proof, a strategy ensuring that Ann reaches $F = 33$ .... **1 point**

(B) Presenting Bob's strategy ensuring that Ann never reaches $F = 35$, with a correct proof that it works .... **3 points**

(B1) Only presenting a correct Bob's strategy preventing $F = 35$, without a correct proof ... **1 point**

(B2) Only presenting, possibly with a proof, Bob's strategy preventing $F = 36$ (or even larger values) **0 points**

Points within one part A or B are not additive. Points from different parts (A and B) are additive.

(X) If no points are awarded for parts A and B, a student may get a following partial credit.

(X1) Introducing the notion of a stable situation (or equivalent), **and** showing either (a) that Ann can increase F whenever the situation before her turn is unstable; or (b) that Bob can preserve the value of F indefinitely, if the situation after some his turn becomes stable; or both (a) and (b) .... **1 point**

Scheme for №6.

1. Full solution of part a): .... **2 points.**

2. Noted that the values of Q in the roots of P are themselves roots of P (not additive with the part a)) **1 point.**