Complete solution of each problem is worth 7 points.

Marking scheme for problem 4

Let $\ell \cap CI = X$, C_1 be the point where AB touches the incircle.

1. Proving that RN touches the incircle or $IX = IN$: 2 points
2. Proving that the triangle XRN is isosceles or $\angle XRI = \angle NRI$: 3 points
3. Proving that R, T, I, M are concyclic, or $MS \parallel IC_1$: 5 points

Items 1, 2, 3 are not additive.

Marking scheme for problem 5

I. Complete solution consists of the following steps.

1. Correct proof of $f(5x) = xf(5)$ for all $x \in \mathbb{Z}$: 2 points
2. Complete proof of $f(x) = xf(1)$ for all x not divisible by 5: 4 points
3. Checking that the functions given by 1. and 2. satisfy the condition: 1 point

This checking should include the reference to the equivalence $5 \mid 4x+3y \Leftrightarrow 5 \mid 3x+y \Leftrightarrow 5 \mid x+2y$, otherwise it is considered incomplete.

Small gaps may lead to 1 or 2 points penalty in any of the steps 1, 2, 3.

II. Partial achievements

1. Proving that $f(-x) = -f(x)$ and (or) $f(2x) = 2f(x)$ for all $x \in \mathbb{Z}$: 1 point

2, Proving the equivalence $5 \mid 4x + 3y \Leftrightarrow 5 \mid 3x + y \Leftrightarrow 5 \mid x + 2y$: 1 point

3. "Five-additivity" $f(5x + 5y) = f(5x) + f(5y)$ obtained: 1 point
4. Conclusion that $f(5x) = xf(5)$: 2 points

5, Small partial achievements, e.g. $f(0) = 0$, $f(-1) = -f(1)$ etc.: 0 points

6. Guessing (without proof) that $f(x) = xf(1)$ for x not divisible by 5 and adequate verification of the obtained function: 1 point

The items 2, 3 are not additive with 4.

Marking scheme for problem 6

Alternating colouring of rows: 0 points

Alternating colouring of rows and correct calculation of the resulting sum: 1 point

Proof that the answer does not exceed $3n^2 + kn + b$ for some $k, b \in \mathbb{Z}$: 1 point