Complete solution of each problem is worth 7 points.

Marking scheme for problem 1

(i) the claim is proved for prime n: 2 points

(ii) the existence of positive d such that $2^d \equiv 1 \pmod{n}$ is declared without the condition $(2,n)=1$: -1 point

(iii) the solution assumes that c or d are less than $\varphi(n)$: -1 point

Marking scheme for problem 2

I. Correct example for $k=2$: 1 point

II. Correct proof of the estimate $k \le 2$: 6 points

Partial achievements in II

II.1. Noting that there are no 3 pairwise disjoint subsets or, equivalently, if subsets A and B are both disjoint with some C then A and B are not disjoint: 1 point

II. 2. Proving that for any disjoint subsets A and B there is unique C intersecting both A and B: 2 points

II.3. Proving the estimate under the assumption that all C_j in the official solution intersect different B_i: 3 points

II.1, II.2 and II.3 are not additive, but any of them is additive with I.

III. The answer without an example: 0 points

IV. Proving that k is even: 0 points

Marking scheme for problem 3

1. Use of Ptolemy's theorem: 0 points

2. The inequality $AC \cdot BD = AB \cdot CD + AD \cdot BC \ge 3\sqrt[3]{AB \cdot CD \cdot \frac{AD \cdot BC}{2} \cdot \frac{AD \cdot BC}{2}}$ or similar for quadrilaterals $BCDE$, $CDEF$, $DEFA$, $EFAB$, or $FABC$: 1 point

3. Proof of the inequalities

3.1. $d_3^2 \ge 64 d_1$: 2 points

3.2. $\sqrt[3]{d_3}\sqrt[6]{d_1} \le \sqrt[3]{d_2} - \sqrt[3]{d_1}$: 2 points

3.3. $\sqrt[3]{d_3^2} \ge \sqrt[3]{d_2} + \sqrt[3]{d_1}$: 2 points

4. Inversion reduced the problem to the inequality $(x+y)(y+z)(z+t)(x+y+z+t) \ge 27xyzt$: 2 points

5. Projective transformation reduced the problem to the case of a hexagon with three pairs of parallel opposite sides: 2 points

6. Use of projective transformations or inversion without getting items 5 or 6: 0 points.

Only parts of item 3 are additive.