THEORETICAL COMPETITION

January 10, 2020

Please read this first:

1. The time available for the theoretical competition is 4 hours. There are three questions.
2. Use only the pen provided.
3. You can use your own calculator for numerical calculations. If you don't have one, please ask for it from Olympiad organizers.
4. You are provided with ***Writing sheet*** and additional paper. You can use the additional paper for drafts of your solutions but these papers will not be checked. Your final solutions which will be evaluated should be on the ***Writing sheets***. Please use as little text as possible. You should mostly use equations, numbers, figures and plots.
5. Use only the front side of ***Writing sheets*****.** Write only inside the boxed area.
6. Begin each question on a separate sheet of paper.
7. Fill the boxes at the top of each sheet of paper with your country (**Country),** your student code (**Student Code**), the question number (**Question Number)**, the progressive number of each sheet **(Page Number**), and the total number of ***Writing sheets*** used (**Total Number of Pages**). If you use some blank ***Writing sheets*** for notes that you do not wish to be evaluated, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, arrange all sheets for each problem in the following order:
    - Used ***Writing sheets*** in order.
    - The sheets you do not wish to be evaluated.
    - Unused sheets.
    - The printed problems.

    Place the papers inside the envelope and leave everything on your desk. You are not allowed to take any paper out of the room.

Problem 1 (10.0 points)

This problem consists of three independent parts.

Problem 1.1 (4.0 points)

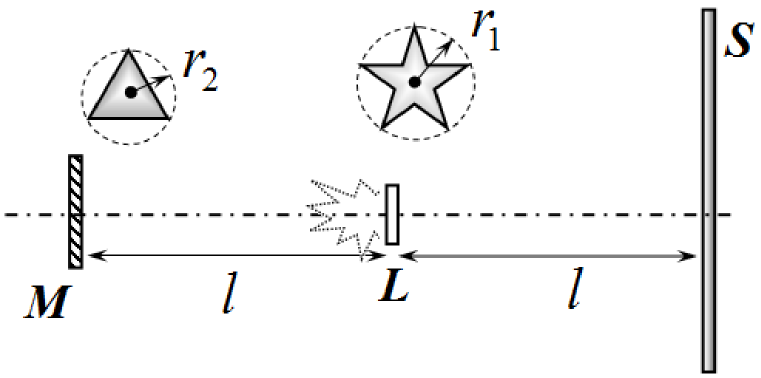
A mathematical pendulum is located on the equator of the Earth. Denote the period of its oscillations at midday T_1, and at midnight T_2. Find the relative difference of these periods $\varepsilon = \frac{T_2 - T_1}{T_1}$. Use the following approximations and numerical values: Earth is an ideal ball of radius $r_1 = 6{,}4 \cdot 10^3\ km$; the Earth's orbit is a circle of radius $r_2 = 1{,}5 \cdot 10^8\ km$ with the center located in the center of the Sun; the axis of rotation of the Earth is perpendicular to the plane of the Earth's orbit; the influence of the Moon and other planets is neglected; acceleration of gravity at the Earth's pole is $g_0 = 9{,}8\ m/s$; the period of revolution of the Earth around its axis is 1 day; the period of revolution of the Earth around the Sun is 1 year.

Problem 1.2 (3.0 points)

The space between two concentric well conductive spheres of radii a and $b > a$ is filled with a substance whose specific resistance depends only on the distance to the spheres' common center. An electric current of strength I flows in between the spheres, such that the bulk density of Joule heat losses in the substance is the same at all points and is equal to w. Determine the average density of the space charge ρ_Q accumulated in the volume of the substance over a sufficiently long time of the electric current flow.

Problem 1.3 (3.0 points)

At equal distances $l = 50\ sm$ in between the screen S and the flat mirror M, a flat matte source L emitting white light only towards the mirror is placed. The planes of the screen, mirror and source are all parallel to one other. The source has the shape of a five-pointed star inscribed in a circle of radius r_1, whereas the mirror has a regular triangle shape inscribed in a circle of radius r_2. The source and mirror centers lie on the same axis perpendicular to the screen plane. Draw a schematic image of the source on the screen, keeping its orientation in accordance with the figure below. Estimate the sizes of all elements of the image.



Consider only two specific cases:

1.3.1 $r_1 = 1{,}0\ mm$ and $r_2 = 10\ mm$;

1.3.2 $r_1 = 10\ mm$ and $r_2 = 0{,}1\ mm$.

Problem 2. Phase states and phase transitions (10.0 points)

At a given pressure, the transition from one phase state of matter to another always occurs at a strictly defined temperature, and the transition itself is called a phase transition. For example, ice at atmospheric pressure melts at 0 ℃, so that when the heat is supplied, the temperature of the mixture of ice and water remains unchanged until all the ice turns into water.

In all the subtasks proposed below, consider that the specific volume of the liquid phase is negligible compared to the specific volume of saturated vapor, which can be considered an ideal gas. Assume as well that the heat capacity of liquid water is independent of temperature.

Useful physical constants

Gas constant $R = 8{,}31\ J/(mol \cdot K)$;
molar mass of air $\mu_{air} = 29{,}0\ g/mol$;
acceleration of gravity $g = 9{,}81\ m/s^2$.
Normal conditions:
pressure $P_0 = 1\ atm = 760\ mm\ Hg = 101325\ Pa$;
temperature $T_0 = 273{,}15\ K = 0$ ℃.

Properties of water (H_2O)

Molar mass of water $\mu_w = 18{,}0\ g/mol$;
water density $\rho_w = 1{,}00\ g/sm^3$;
ice density $\rho_i = 0{,}920\ g/sm^3$;
melting point of ice at normal pressure $t_m = 0{,}00$ ℃;
boiling point of water at normal pressure $t_b = 100{,}0$ ℃;
specific heat of water $c_w = 4{,}20\ J/(g \cdot K)$;
specific heat of melting ice $q_i = 334\ J/g$;
specific heat of water vaporization (at 100 ℃) $r_w = 2259\ J/g$;
Poisson's adiabatic exponent for water vapor $\gamma = C_P/C_V = 4/3$.

Specific heat of phase transition

If a transition from one phase state to another is associated with the release or absorption of a certain amount of heat, called the transition heat, then such a transition is called the first-order phase transition. In this case, the transition heat q for a unit mass is called the specific heat of the phase transition (melting, evaporation, sublimation).

Since the phase transition occurs at constant pressure, according to the first law of thermodynamics, the heat q is spent on changing the internal energy u and on performing the work A against constant external pressure:

$$q = u_2 - u_1 + A,$$

where u_1, u_2 stand for the specific internal energies of the first and second phases, respectively.

During melting (crystallization), due to a small difference in the densities of the liquid and solid phases, the volume change as a result of the phase transition is small, therefore, the work A can be neglected in comparison with the change in internal energy.

2.1 Evaluate how much of the evaporation heat of water at $t_b = 100$ ℃ is spent on changing the internal energy. Express your answer in %.

2.2 Evaluate the specific heat of water vaporization at room temperature $t = 20{,}0$ ℃.

In the following, the specific heat of vaporization of all liquids is considered to be temperature independent.

The Clausius–Clapeyron relation

When the pressure changes, the temperature of the first-order phase transition changes as well, i.e. the phase transition occurs at a strictly defined dependence $P(T)$ between the pressure P and the temperature T of the matter under investigation. This dependence, depicted on the (T,P)-plane, is called the (T,P)-phase diagram, and the $P(T)$ curve itself is called the phase equilibrium curve. The Clapeyron-Clausius relation gives the slope of the phase equilibrium curve $P(T)$ in the following form:

$$\frac{dP}{dT} = \frac{q}{T(v_2 - v_1)},$$

where q denotes the specific heat of transition from phase 1 with the specific volume v_1 to phase 2 with the specific volume v_2.

2.3 Assuming that the pressure of the saturated water vapor at the temperature $t_b = 100\ ℃$ is known, obtain an explicit dependence of the pressure of the saturated water vapor on its temperature $P(T)$.
2.4 Evaluate the boiling point of water at the highest peak in Kazakhstan – Khan-Tengri mountain. The height of the Khan-Tengri mountain peak is $h \approx 7000\ m$ above the sea level. The altitude air temperature should be considered constant and equal to $t_0 = 0$ ℃.
2.5 At what pressure (in atmospheres) the ice melts at the temperature of $t = -\ 1{,}00$ ℃?
2.6 It is known that ice crystals begin to break down if a force is applied along any direction of the crystal to create a pressure $P > P_{cr} \sim 1000\ atm$. Therefore, snow in frosty weather "crunches" when walking. Estimate the maximum air temperature t_{max} at which the snow still “crunches” when walking.
2.7 One mole of the saturated water vapor occupies a vessel and has the temperature of $t_b = 100\ ^0$C. The vapor heats up and at the same time its volume changes such that it remains saturated at all times. Find the molar heat capacity of vapor in such a process.

Border boiling

Border boiling is boiling at the interface between two immiscible liquids. The border boiling point may vary significantly from the volume boiling points of each liquid.

Tetrachloromethane or hydrogen tetrachloride is a heavy (density $\rho = 1{,}60\ g/sm^3$) transparent liquid with a molar mass $\mu = 153{,}8\ g/mol$. Under normal atmospheric pressure, carbon tetrachloride boils at a temperature of $t = 76{,}65$ ℃, while it practically does not dissolve in water. A vessel with a volume of $V = 100\ ml$ is half-filled with the carbon tetrachloride, and the same (by volume) amount of water is poured over it. In this case, a clear water-carbon tetrachloride border is formed. When the vessel is uniformly heated in a water bath, the border boiling at the liquid interface begins at the temperature of $t^* = 66{,}0$ ℃, which is significantly lower than the volume boiling temperature of each liquid.

2.8 Calculate the specific heat of evaporation of carbon tetrachloride, if it is known that the pressure of the saturated water vapor at the border boiling point is $P_w(t^*) = 196\ mm\ Hg$.
2.9 Find the mass of liquid remaining in the vessel by the time the other liquid is completely boiled away at such border boiling.

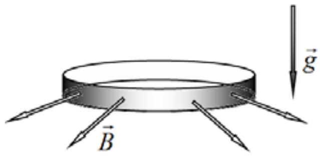
Consider another pair of immiscible liquids, water and fluoroketone.

Fluoroketone, sometimes called “dry water”, is used to extinguish fires in libraries, museums, and offices because it does not wet paper. It is a heavy (density $\rho = 1{,}72\ g/sm^3$) transparent liquid with a molar mass $\mu = 316\ g/mol$, which practically does not dissolve in water. The boiling point of fluoroketone at atmospheric pressure is $t_f = 49{,}2$ ℃, its specific heat of vaporization is $r = 95{,}0\ J/g$. If water is poured over the fluoroketone into the vessel, a clear water-fluoroketone border is also formed.

2.10 Estimate the boiling point t_x of liquids at the water-fluoroketone border if the saturated vapor pressure of water is known at the volume boiling point of fluoroketone to be $P_w(t_f) = 89{,}0\ mm\ Hg$.

Problem 3. Ring in a magnetic field (10.0 points)

Uniformly charged ring



A very thin ring of mass m and radius r is uniformly charged along its length with a charge q. At the initial moment of time, the ring rests horizontally and is released without a push. The subsequent motion of the ring appears in the vertical gravitational field of the Earth, characterized by the acceleration of gravity g and in the horizontal radial magnetic field of induction B. Neglect air resistance and assume that the plane of the ring remains horizontal at all times.

3.1 Find the maximum velocity of the ring center of mass v_{max} for the entire time of motion.

3.2 Find the time interval Δt elapsed from the start of the ring motion to its first reaching of the maximum velocity of the center of mass.

3.3 Find the maximum height h_{max} at which the ring center of mass falls over the entire time of motion.

Conductive ring

A very thin ring of mass m and radius r is made of a conductive material with a resistivity ρ and a cross section area $s \ll r^2$. At the initial time moment $t = 0$ the ring rests horizontally and is released without a push. The subsequent motion of the ring appears in the vertical gravitational field of the Earth, characterized by the acceleration of gravity g and in the horizontal radial magnetic field of induction B. Neglect air resistance and assume that the plane of the ring remains horizontal at all times.

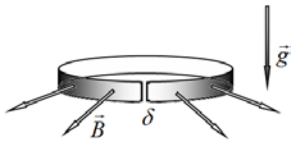
3.4 Find the steady-state velocity v_0 of the ring center of mass after a sufficiently large period of time having passed.

3.5 The dependence of the current strength $I(t)$ in the ring on time t has the following form

$$I(t) = A_1 + B_1 \exp(\gamma_1 t).$$

Find the constants A_1, B_1 and γ_1.

Conductive ring with a cut



A very thin ring of mass m and radius r is made of a conductive material with a resistivity ρ and a cross section area s. A cut with a width $\delta \ll \sqrt{s} \ll r$ was made along the radius of the ring. At the initial time moment $t = 0$ the ring rests horizontally and is released without a push. The subsequent motion of the ring appears in the vertical gravitational field of the Earth, characterized by the acceleration of gravity g and in the horizontal radial magnetic field of induction B. Neglect air resistance and assume that the plane of the ring remains horizontal at all times.

3.6 Find the steady-state acceleration a_0 of the ring center of mass after a sufficiently large period of time having passed.

3.7 The dependence of the current strength $I(t)$ in the ring on time t has the following form

$$I(t) = A_2 + B_2 \exp(\gamma_2 t).$$

Find the constants A_2, B_2 and γ_2.

Mathematical hints for the theoretical problems

The following integrals may be useful:

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b|.$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}, \text{ where } n \text{ is integer}$$

$$(1+x)^\gamma \approx 1 + \gamma x + \frac{\gamma(\gamma-1)}{2} x^2, \textit{ for } x \ll 1 \textit{ and any } \gamma$$