THEORETICAL COMPETITION

January 13, 2010

Please read this first:

1. The time available for the theoretical competition is 4 hours. There are three questions.
2. Use only the pen provided.
3. You can use your own calculator for numerical calculations. If you don't have one, please ask for it from Olympiad organizers.
4. You are provided with ***Writing sheet*** and additional paper. You can use the additional paper for drafts of your solutions but these papers will not be checked. Your final solutions which will be evaluated should be on the ***Writing sheets***. Please use as little text as possible. You should mostly use equations, numbers, figures and plots.
5. Use only the front side of ***Writing sheets*.** Write only inside the bordered area.
6. Begin each question on a separate sheet.
7. Fill the boxes at the top of each sheet of paper with your country (**Country),** your student code (**Student Code**), the question number (**Question Number)**, the progressive number of each sheet **(Page Number**), and the total number of ***Writing sheets*** used (**Total Number of Pages**). If you use some blank ***Writing sheets*** for notes that you do not wish to be evaluated, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, arrange all sheets for each problem in the following order:
    - Used ***Writing sheets*** in order;
    - The sheets you do not wish to be evaluated
    - Unused sheets and the printed question.

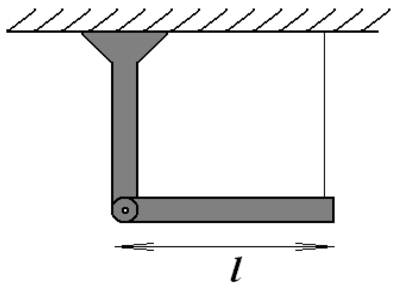
    Place the papers inside the envelope and leave everything on your desk. You are not allowed to take any paper out of the room.

Problem 1 (10 points)

This problem consists of three unrelated parts.

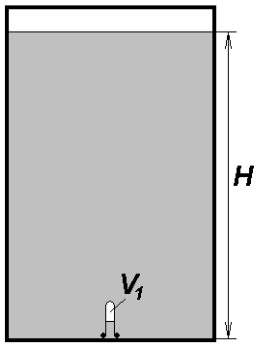
1A (3 points)

One end of a homogeneous rigid rod with mass m and length l is suspended on the vertical support by an ideal joint, and the other end hangs on a thread such that the rod is in horizontal position. At a certain time point the thread is cut. Find the dependence of reaction force of the joint on the angle α of the rod deviation from the horizontal position.



1B (4 points)

Liquid is poured in a closed cylindrical vessel with thick walls. The height of the liquid level is H. The mass density of the liquid decreases linearly with the height from ρ_{max} to small value which can be assumed to be equal to zero. Thin layer of vapor is saturated over the liquid surface but its pressure can be neglected with respect to the hydrostatic pressure of the liquid. Inverted test-tube of the volume V_0 is placed at the bottom of the vessel. The mass M of the test-tube is concentrated at its neck, and its length is small compared to H. The gas of volume V_1 with negligible mass is placed inside the test-tube. The temperature of the system is kept constant. Can the test-tube stay motionless in the liquid at a certain height from the bottom? If it is possible, what conditions are to be satisfied? What is the height of the test-tube position over the bottom in this case? Is this state stable? The volume of walls of the test-tube is small compared to V_1.



1C (3 points)

Rectangular light source of size *c* × *b* is fixed on the ceiling of the room of height *L = 3.0 m.*

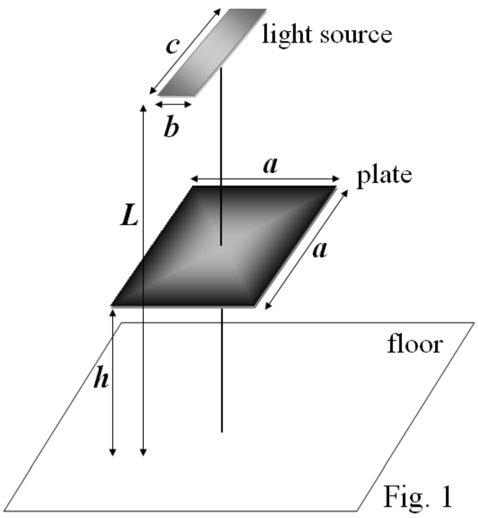


Fig. 1

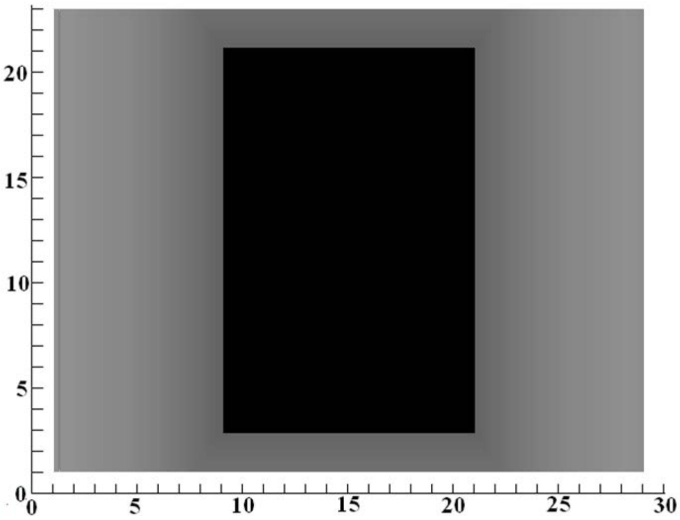
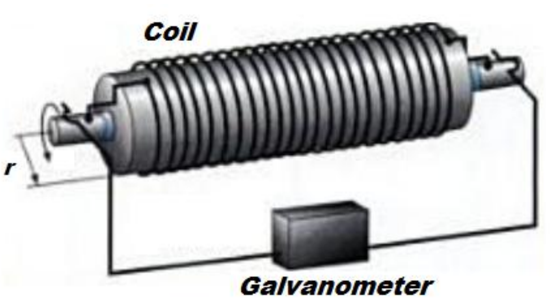


Fig. 2

Opaque square plate with side *a = 10 cm* is placed horizontally under the lamp at a height *h* above the floor (Fig. 1). Fig. 2 schematically shows the shape of the shadow casted by the plate on the room floor. The shadow consists of a dark rectangle surrounded by a lighter rectangle, and when approaching the edges of the outer rectangle the shade turns lighter. The value of the scale division in Fig. 2 is *1.0 cm*. Find the sizes of the light source *b* and *c*, and the height *h* of the plate position. Specify the exact orientation of the light source with respect to the plate shadow.

Problem 2 (10 points)
The Tolmen-Stewart experiment



In 1916 Tolmen and Stewart carried out their famous experiment proving that the electric current in metals was caused by freely moving electrons. The sketch of the experimental setup is shown on the left hand side.

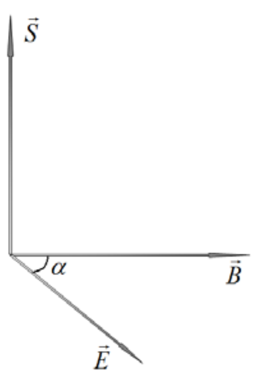
A long coil of the radius r and the length h has an inertia moment J_0. The coil is reeled up by a single layer of the metal wire of the length ℓ and mass m such that the number of loops on the unit length equals n. The both ends of the wire are abridged to the galvanometer by the sliding contacts. The coil together with the wire is set rotating with the angular velocity ω_0 and, then, the friction force with the torque M is applied to stop the coil rotation. The total resistance of the circuit is R and its capacity is assumed negligible.

Part 1. In this part of the problem you can neglect the inductance of the coil and assume that the Ohm's law holds at any time.

1. **[1 point]** Determine the total inertia moment J of the coil together with the wire. Express your answer in terms of J_0, m, r.
2. **[1 point]** Determine the dependence of the angular velocity $\omega(t)$ of the coil on time t. Express your answer in terms of J, M, ω_0.
3. **[1 point]** Determine the dependence of the electric current strength $I(t)$ on time t. Express your answer in terms of J, M, r, ℓ, R with m_e, e being the mass and the electric charge of electron, respectively.
4. **[2 points]** During the experiment the galvanometer has registered the electric charge Q passed in the circuit. Express the charge-to-mass ratio of the electron e/m_e in terms of ω_0, r, ℓ, R, Q.

Part 2. In this part of the problem you have to take into account the small inductance of the coil which is supposed to be quite long to neglect the side effects.

5. **[1 point]** Find the maximal electric current strength $I_{\max}$ in the coil. Express your answer in terms of J, M, r, ℓ, R, m_e, e. Plot a qualitative dependence $I(t)$.
6. **[1 point]** Find the maximal energy W_0 stored in the coil during the experiment. Express your answer in terms of $J, M, r, \ell, R, m_e, e, n, h$ and the magnetic constant μ_0.
7. **[3 points]** The flux S of the electromagnetic energy through the unit area is determined by the Pointing vector which is perpendicular to both electric and magnetic fields with the module $S = \dfrac{1}{\mu_0} EB \sin\alpha$, where E is the electric field strength vector, B is the magnetic induction vector and α is the angle between them (see the picture on the right). Find the electromagnetic energy W passing through the lateral surface for the time period while the electric current increases and the electromagnetic



energy W' passing through the lateral surface for the time period while the electric current decreases. Express your answer in terms of $\ell, r, M, J, n, R, m_e, e, \mu_0$.

Problem 3 (10 points)
The relic radiation and cosmic rays

According to modern cosmology, our universe is filled with electromagnetic radiation, remained after the Big Bang and called the cosmic **relict** radiation, or cosmic microwave background (shortly, CMB). With good accuracy, the CMB is homogeneous and isotropic in the reference system associated with our Galaxy which we take as the laboratory reference system. The frequency distribution of the CMB coincides with the spectrum of a blackbody at temperature $T = 2.7\ K$. Discovery and study of properties of the CMB were awarded Nobel Prizes in Physics in 1978 and 2006.

Relict photons are extremely numerous and, therefore, may affect Galaxy radiation of another nature called **cosmic rays** (shortly, CR). It is believed that CR are formed in stellar explosions. CR consist mainly of protons whose energy can exceed by many orders the energy of terrestrial accelerators. The mechanism of generation of ultrahigh energy CR is not entirely clear, but the experimentally observed energy distribution of CR is limited above by the energy $E_p^{max} = 10^{21}\ eV$. In this problem it is assumed that this limitation is caused by energy loss in the process of interaction between protons and the CMB. Protons can participate in the Compton scattering

$$p + \gamma \rightarrow p + \gamma \qquad (1)$$

and cause the reaction

$$p + \gamma \rightarrow \Delta \rightarrow \pi^0 + p \qquad (2)$$

where p is a proton, γ is a relict photon, Δ is the lightest, as compared to the nucleon, baryon with the rest mass $m_\Delta = 1.232 \times 10^6\ eV/c^2$, which quickly decays into pi-meson π^0 and proton p. (Δ-particle decays also to π^+-meson and neutron. Neutron quickly turns into proton by β-decay, so the consideration of this reaction channel is not essential in this Problem). Due to the birth of Δ-particles, the probability of interaction of protons with γ-quanta increases dramatically.

The purpose of this Problem is to determine the upper limit of the observed energy spectrum of CR, supposing that reaction (2) is the main mechanism of energy loss of CR, and compare the loss of energy protons in the reactions (1) and (2).

In the following questions, you have to measure the energy in electron-volts eV, and momenta in eV/c, where c is the speed of light in vacuum. The rest mass of the proton is $m_p = 938 \times 10^6 eV/c^2$, the rest mass of pi-meson is $m_\pi = 140 \times 10^6 eV/c^2$. Boltzmann constant is $k = 1.38 \times 10^{-23} Дж \cdot K^{-1}$.

1. Estimate most probable energy E_γ and the corresponding momentum p_γ of relict photons, given that they correspond to the blackbody radiation at temperature $T = 2.7\ K$.

It is assumed in the following questions that the initial photon in reactions (1) and (2) has the energy and the momentum found Question 1.

2. The rest mass m of a particle is related to the total relativistic energy E and momentum $\vec{p}$ of this particle in an arbitrary inertial reference system as $E^2/c^2 - \vec{p}^{\,2} = m^2c^2$. Here, the quantity mc^2 does not depend on the reference system and is a complete internal energy of the particle. Write a corresponding expression for the total internal energy of a system of two non-interacting particles (i.e. the total energy in the reference frame in which the total

momentum of physical system is equal to zero) having the total energies E_1, E_2 and momenta $\vec{p}_1$, $\vec{p}_2$.

For further analysis you may need the relativistic law of transformation of momentum and energy of the particle. In the transition from the inertial reference system S to an inertial system S', moving along the positive direction of the axis Z ($OZ \uparrow\uparrow OZ$) with the speed V_0 in the reference system, energy and momentum transform like the coordinates of space-time point $(x, y, z, t) \rightarrow (x', y', z', t')$:

$$p_z c = G(p_z' c + \frac{V_0}{c} E'),$$
$$p_x = p_x',$$
$$p_y = p_y',$$
$$E = G(E' c + \frac{V_0}{c} p_z' c). \qquad (3)$$

Here,

$$E'/c = \sqrt[2]{m^2c^2 + p_x'^2 + p_y'^2 + p_z'^2}, \quad G = 1/\sqrt[2]{1-(V/c)^2}.$$

3. Determine the lowest possible energy of the proton at which reaction (2) turns possible:
   a) in the center of mass of proton and photon $p + \gamma$ (i.e., the reference system in which the total momentum of the proton and photon is zero);
   b) in the Galaxy system of reference.

Starting from the obtained results, determine the maximum energy of protons in cosmic rays.

4. Assuming that in the Galaxy system a proton has the maximum energy E_p^{max} and commits a head-to-head collision with a relict photon, determine the momentum of pi-meson π^0 in reaction (2) in this frame of reference for the case when the emitted particles move along or against the direction of the initial momentum of the proton. What is the change of the momentum of the proton in this case?

5. What is the value of momentum of the outgoing photon in the Compton scattering (1) under the same conditions as in the Question 4?

6. Is reaction (2) with the cosmic relict radiation possible, if the initial momenta of the proton and the photon are parallel in the Galaxy system? If the reaction is possible, what is the minimum value of the photon momentum in the Galaxy system?