

Complete solution of each problem is worth 7 points.

Marking scheme for problem 1

(1) P, Q, R, S are concyclic: 3 points

(2) $PM = QM$: 1 point

(3) $MS = MR$: 3 points

Points are given only when the statements are proved. Points for (1) and (2) are not added.

Marking scheme for problem 2

Trivial observations such as $f(0) = 0$, or $f(x) \equiv 0$ and $f(x) \equiv x$ are solutions: 0 points

I. Case $f(x) = 0$ iff $x = 0$

a) Proving that $f(x) \equiv x$: 2 points

b) Partial achievement $f(-x^2) = -x^2$: 1 point

Note: a) and b) are not added

II. Case $f(x_0) = 0$ for some nonzero x_0

1. Full solution for this case: 5 points

2. Deduction of points:

a) One of two subseries $f(x) = \begin{cases} 0, x \neq b \\ -\sqrt{b}, x = b \end{cases}$, $f(x) = \begin{cases} 0, x \neq b \\ -\sqrt{-b}, x = b \end{cases}$ is omitted: -2

points

b) verification of $f(x)$ given by $f(x) = \begin{cases} 0, x \neq -a^2 \\ a, x = -a^2 \end{cases}$ is omitted or failed: -2 points

c) only the case $x \neq -a^2$ is verified: -1 point

d) other minor mistakes: -1 point

Partial achievements:

a) $f(y^2) = 0$ for all y : 1 point

b) at least one solution given by $f(x) = \begin{cases} 0, x \neq b \\ -\sqrt{b}, x = b \end{cases}$, $f(x) = \begin{cases} 0, x \neq b \\ -\sqrt{-b}, x = b \end{cases}$ or

$f(x) = \begin{cases} 0, x \neq -a^2 \\ a, x = -a^2 \end{cases}$ is found: 1 point

c) useful application of the substitution $y \rightarrow -y$, for example obtaining of formula (2) in the official solution: 1 point

d) $x \neq \pm\sqrt{-x_0}f(x_0) \dots$ (or equivalent) $\Rightarrow f(x) = 0$: 1 point

Note: Only a) and b) or a) and d) may be added

Marking scheme for problem 3

Claim that a regular colouring (defined in any recognizable way) exists: 1 point