XIV International Zhautykov Olympiad in Mathematics
Almaty, 2018

January 13, 9.00-13.30
Second day
(Each problem is worth 7 points)

4. The Crocodile thought of four unit squares of a 2018×2018 forming a rectangle with sides 1 and 4. The Bear can choose any square formed by 9 unit squares and ask whether it contains at least one of the four Crocodile's squares. What minimum number of questions should he ask to be sure of at least one affirmative answer?

5. Find all real a for which there exists a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x - f(y)) = f(x) + a[y]$ for every real x и y ($[y]$ denotes the integral part of y).

6. A convex hexagon $ABCDEF$ is inscribed in a circle with radius R. Diagonals AD and BE, BE and CF, AD and CF of the hexagon meet at points M, N, K respectively. Let r_1, r_2, r_3, r_4, r_5, r_6 be the inradii of the triangles ABM, BCN, CDK, DEM, EFN, AFK respectively. Prove that $r_1 + r_2 + r_3 + r_4 + r_5 + r_6 \leqslant R\sqrt{3}$.