SOLUTION TO THE EXPERIMENTAL COMPETITION

The Law of Archimedes (15.0 points)

Part 1. Installation parameters

1.1 A strip of millimeter paper is screwed onto the test-tube. We make marks on the strip, untwist it and obtain the lengths of 1, 2, 3 and 4 revolutions as

$l_1 = 63 mm$

$l_2 = 127 mm$

$l_3 = 191 mm$

$l_4 = 255 mm$

From these data we find that the length of one revolution is equal to $\langle l \rangle = (64,0 \pm 0.3) mm$

The diameter is then calculated by the formula $D = \frac{\langle l \rangle}{\pi} = 20,372 mm$, the unstrumental error is found as $\Delta D = D\frac{\Delta l}{\langle l \rangle} = 0,1 mm$ and the final result is written as

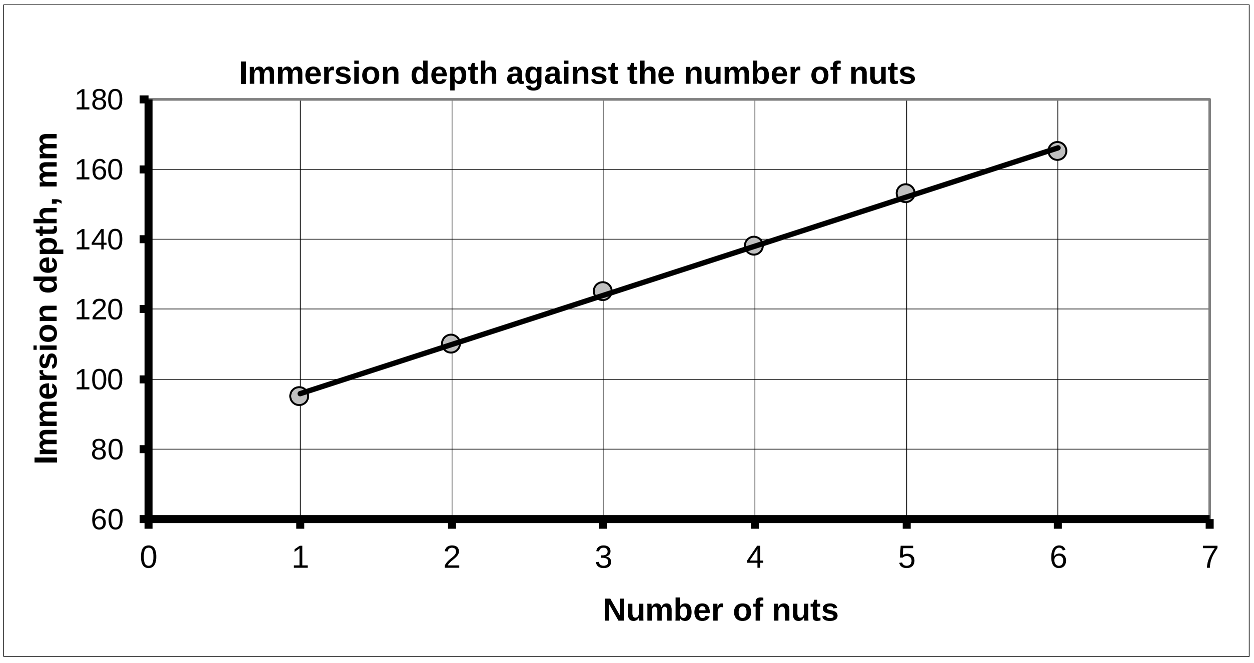
$$D = (20,4 \pm 0.1) mm.$$

1.2 The length of the test-tube is obtained as $L = (175 \pm 1) mm.$

1.3.1 – 1.3.2 . Dependence of the immersion depth of the tesr-tube on the number of nuts, placed in it, is shown in Table 1. First, the length x of the part of the test-tube, protruding above the water level, is measured. And then the immersion depth is calculated by the formula $h = L - x$.

Table 1

Number of nuts	x, mm	h, mm
1	80	95
2	65	110
3	50	125
4	37	138
5	22	153
6	10	165



The dependence obtained is linear and is described by the formula

$$h = an + b. \tag{1}$$

The parameters, calculated by the least square method, are equal

$$\begin{aligned} a &= (14{,}1 \pm 0{,}5)mm \\ b &= (81{,}8 \pm 1{,}8)mm \end{aligned}. \tag{2}$$

1.3.3 The theoretical formula for the resulting dependence follows from the equilibrium condition

$$(M + mn)g = \rho Shd \quad \Rightarrow \quad h = \frac{M + mn}{\rho S}. \tag{3}$$

where $S = \frac{\pi D^2}{4}$ stands for the cross-sectional area of the test tube.

From the comparison of expressions (3) and (1) it follows that

$$a = \frac{m}{\rho S} \quad \Rightarrow \quad m = \rho S a . \tag{4}$$

Numerical calculations lead to the following result

$$m = \rho \frac{\pi D^2}{4} a = 4{,}58 \cdot 10^{-3} kg = 4{,}58\ g. \tag{5}$$

The instrumental error in measuring the mass of the nut is calculated by the formula

$$\Delta m = m \sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(2\frac{\Delta D}{D}\right)^2} = 1{,}6 \cdot 10^{-4} kg. \tag{6}$$

The final weight of the nut is written as

$$m = (4{,}58 \pm 0{,}16)\, g. \tag{7}$$

The weight of the test-tube is calculated by the formula

$$b = \frac{M}{\rho S} \quad \Rightarrow \quad M = \rho S b = \rho \frac{\pi D^2}{4} b = 2{,}67 \cdot 10^{-2} kg = 26{,}7 g.$$

The error in calculating the mass of the test-tube is found as

$$\Delta M = M \sqrt{\left(\frac{\Delta b}{b}\right)^2 + \left(2\frac{\Delta D}{D}\right)^2} = 0{,}6 g.. \tag{8}$$

To simplify further calculations, we note that the ratio of the parameters of the linear dependence (2) is equal to the ratio of the mass of the test-tube and the nut:

$$n^* = \frac{M}{m} = \frac{b}{a} = 5{,}82 . \tag{9}$$

Part 2. Oscillations of the test-tube

2.1 To simplify the calculations, the formula for the period of oscillations can be rewritten in the form

$$T_n = 2\pi \sqrt{\frac{h_0}{g}} = 2\pi \sqrt{\frac{an + b}{g}} . \tag{10}$$

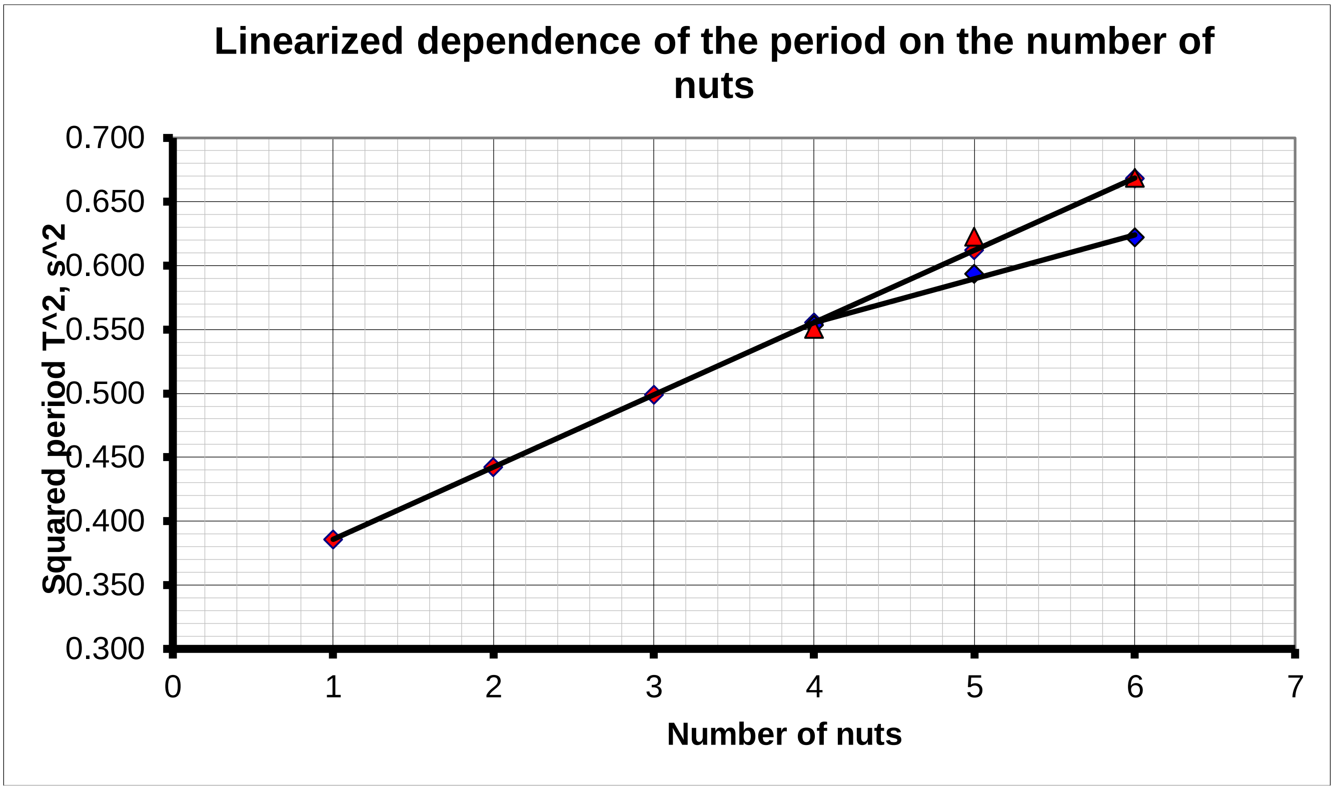
To linearize this dependence, it is necessary to plot and analyze the dependence of the squared period on the number of nuts $T^2(n)$. The results are summarized in Table 2.

Table 2.

Number of nuts	T, s	T^2, s^2
1	0,680	0,463
2	0,717	0,514
3	0,752	0,565

4	0,785	0,617
5	0,817	0,668
6	0,848	0,720

The graph of the dependence $T^2(n)$ is shown in the figure below.



2.2 The results of the measurements are given in tables

The random error in measuring the period is estimated from the following formula

$$\Delta t = 2\sqrt{\frac{\sum_k (t_k - \langle t\rangle)^2}{N(N-1)}} \quad ; \qquad \Delta T = \frac{\Delta t}{k}. \qquad (11)$$

Here t refers to the time needed to perform k periods of oscillations (in our case $k = 5$ and $k = 3$ respectively), $N = 10$ stands for the number of measurements.

Table 3. Oscillations in the wide vessel

Number of nuts	Number of periods k	Time t, s	Period T, s	Averaged period $\langle T\rangle$, s	Error in the period ΔT	Squared period T^2, s^2
4	5	3,74	0,748	0,744	0,009	0,554
	5	3,64	0,728			
	5	3,77	0,754			
	5	3,71	0,742			
	5	3,74	0,748			
5	5	3,93	0,786	0,770	0,010	0,594
	5	3,81	0,762			
	5	3,89	0,778			
	5	3,83	0,766			
	5	3,80	0,760			

6	5	3,93	0,786	0,789	0,010	0,622
	5	3,93	0,786			
	5	4,04	0,808			
	5	3,93	0,786			
	5	3,89	0,778			

Table 3. Oscillations in the beaker

Number of nuts	Number of periods k	Time t ,s	Period T , s	Averaged period $\langle T \rangle$, s	Error in the period ΔT	Squared period T^2, s^2
4	3	2,21	0,74	0,742	0,014	0,551
	3	2,25	0,75			
	3	2,27	0,76			
	3	2,20	0,73			
	3	2,20	0,73			
5	3	2,38	0,79	0,789	0,015	0,623
	3	2,37	0,79			
	3	2,34	0,78			
	3	2,42	0,81			
	3	2,33	0,78			
6	2	1,61	0,81	0,818	0,036	0,669
	2	1,59	0,80			
	2	1,66	0,83			
	2	1,63	0,82			
	2	1,69	0,85			

2.4 What possible reasons can explain the deviation between experimental data and theoretical calculations?

Table 4

No.	**Possible reasons**	**«Yes»**	**«No»**
1	Measurement errors	X	
2	Oscillation damping		X
3	An increase in the effective mass of a moving test-tube due to water entraining		X
4	Change in pressure under the tube when it moves as compared to hydrostatic pressure	X	
5	Surface tension forces		X

Comments:
1. Of course, errors ifluence any result.
2.3 These reasons should lead to an increase in the period, and not to a decrease.
4. Apparently, the main reason, leading to a reduction in the period.
5. Too small forces.

Marking scheme

Part1. Installation parameters

№	Criteria	Total	Points
1.1	**Diamater measurement**	**0,9**	
	- sketch of the measurements: - rolling on the test-tube (2-3 revolutions; *1 revolution*); *- rolling the test-tube on the millimeter paper;* *- direct measurement of the diameter;*		 0,2 (*0,1*) (*0,1*) (*0,1*)
	Measurement results: - circumference in the range of 63-66 mm (*61-68 mm, out of range*)		0,2 *(0,1; 0)*
	Evaluation of the diameter: - formula: - numerical value (in accordance with the previous part)		 0,1 0,2 *(0,1; 0)*
	Instrumental errpr 0,25-0,35 mm (*larger*)		0,1 *(0)*
	Correctly rouded results*		0,1
1.2	**Measurement of the test-tube length**	**0,3**	
	- length in the range of 170-180 mm (*out of range*) - instrumental error 1 mm (*иное*)		0,1 *(0)* 0,1 *(0)*
	Correctly rounded result*		0,1
1.3.1	**Results of the immersion depth measurement**	**1,8**	
	Results differ from tabulated ± 2 mm (± 4 *mm, larger*)		1,2 *(0,6; 0)*
	Number of points* 6 (*3, less*)		0,6 *(0,3, 0)*
1.3.2	**Plotting the graph and calculating the parameters of the dependence** (marked only if 1.3.1 has been marked)	**1,0**	
	- axes are signed and ticked; - points are plotted in accordance with the table		0,1 0,2
	Parameters of the dependence: - form of dependence is a linear function - evaluation of the parameters; - errors of the parameters;		 0,1 2x0.2 2x0,2
1.3.3	**Calculation of masses of the nut and the test tube:** (marked only if 1.3.1 has been marked)	**2,0**	
	- formula of the theoretical dependence		0,4
	- formulas for calculating masses through the parameters of the linear dependence;		2x0,2
	- calculation of the mass of the nut: within 10% from the tabulated value (20%, larger)		0,4 *(0,2, 0)*
	- Nut mass error: errors in the slope and the diameter are taken into account (*only one contribution)*		0,2 *(0,1)*
	- calculation of the test-tube mass: within 10% from the tabulated value (20%, larger)		0,4 *(0,2, 0)*
	- error in the mass of the test-tube: errors in the shift and the diameter: erroes in the shift and in the diameter are taken into account (*only one contribution)*		0,2 *(0,1)*

* - marked only if the measurements are marked.

Part 2. Oscillations of the test-tube

№	Criteria	Total	Points
2.1	**Theoretical dependence**	**1,2**	
	- formula for the period $T(n)$ via measured parameters		0,2
	- periods are calculated		6x0,1
	- linearization $T^2(n)$ *(other)*		0,1(0)
	Plotting the graph: - axes are signed and ticked; - points are plotted in accordance with the table;		 0,1 0,2
2.2	Formula for evaluating the error in the period: - decrease of the random error with increasing the number of measurements; *- modulus of the average deviation from the mean value;*		 0,2 *(0,1)*
	Oscillations in the wide vessel	**3,0**	
	Results within the range $\pm 20\%$ ($\pm 30\%$, larger)		3x0,3 (0,2; 0)
	More than 7 measuments are taken (more than 4, less)*		3x0,3 (0.2; 0)
	Periods are calculated*		3x0,1
	Errors are calculated*		3x0,1
	Points are plotted in accordance with the table *		0.2
	Errors are stated in the graph*		0,2
	The periods of oscillations are found to be less than the theoretical one (more than 0,1 s)*		0,2
	Oscillations in the beaker	**3,3**	
	The results of the measurements within the range $\pm 20\%$ ($\pm 30\%$, larger)		3x0,3 (0,2; 0)
	More than 7 measuments are taken (more than 4, less)*		3x0,3 (0.2; 0)
	Periods are calculated*		3x0,1
	Errors are calculated*		3x0,1
	Points are plotted in accordance with the table *		0.2
	Errors are stated in the graph*		0,2
	The periods of oscillations are close to theoretical (the difference is not more than 0,2 s)*		0,3
	The periods of oscillations in different vessels are similar (differences not more than 0.2 s) *		0,2
2.4	Possible reasons	**1,5**	
	- each correct answer		5x0,3
	Total	**15**	

* - marked only if the measurements are marked.