Problem 1 (10 points)

This task consists of three parts which are not related to each other.

Task 1.A. 2012 (4 points)

One end of the rigid weightless rod is pivotally attached. To the other end by the thread thrown over the weightless block, mass m is hung. Two more masses of 20m and 12m are hung by the threads to the rod at the points that divide it into three equal parts (see Fig.). All the threads are inextensible and weightless. The rod is hold in a horizontal position and then it is released. Find the acceleration of all the weights immediately after the release of the rod.

Task 1.B. And diodes ... (2.5 points)

On a separate form you can find a graph of the current-voltage characteristics of one diode (dependence of the current through the diode on voltage on it). Five of same diodes are connected as shown in Figure. Plot a graph of the current in the circuit of the voltage source U, if the latter varies from 0 to 3 V.

To plot, use a separate form given to you.
Do not forget to return it!

Task 1.C. A flat lens (3.5 points)

Round transparent plane-parallel plate of the thickness h is made from the material which optical refractive index depends on the distance r to the central axis of the plate according to the law

(1)

where n0, beta are known positive constants. The plate is in the air whose refractive index n=1.
On the axis of the plate at a distance a (a<<h) the point light source S is places. Show that the plate is a kind of lens, that is, it forms an image of the source. Determine at what distance from the plate the image of the source occurs. What is the focal length of this lens?

Problem 2

Adventures of a piston (10 points)

An open cylindrical vessel of the height H=30.0 cm and cross-sectional area S=50.0 cm2 is filled by air under normal conditions, i.e. at the atmospheric pressure p0=1.01 x 10 5 Pa and the temperature T=273 K. The thin top heavy piston of mass M=50.0 kg is gently inserted into the vessel. The vessel wall and the piston are made of a material that conducts heat very poorly. Assume that air is an ideal diatomic gas with an average molar mass mu=29.0 g/mol, the acceleration of free fall is g= 9.80 m/s2, and the universal gas constant is equal to R= 8.31 J/(mol K). Heat capacity of the piston and the vessel, as well as the friction on the walls of the piston is completely ignored.

The piston is released. The process of transition to the final balance is done in two stages. In the first stage the piston oscillates. These gas processes cannot be considered in equilibrium. Because of the non-equilibrium, fluctuations of the piston are damped, i.e. mechanical energy is dissipated. Consider that half of the dissipated energy is transferred to the gas in the vessel and the other half to the atmosphere. At this stage it is also possible to neglect the thermal conductivity of the vessel and the piston. After the oscillations stop, the piston stops at some height H1.

The second stage is slow, i.e. during some period of time the piston moves and finally stops at the some height H2.

2.1 [0.5 points] What is the air pressure p1 in the vessel at the end of the first stage? Express the reply in terms of atmospheric pressure p0, adiabatic index gamma, and the parameter alpha. Find the numerical value of p1.
2.2 [1.5 points] What is the temperature T1 at the end of the first stage? Express the reply in terms of T0, gamma, and alpha. Find the numerical value of T1.
2.3 [0.5 points] Find the height H1. Express reply in terms of H, gamma, and alpha. Find the numerical value of H1.
2.4 [0.5 points] What is the air pressure p2 in the vessel at the end of the second stage? Express the reply in terms of p0 and alpha. Find the numerical value of p2.
2.5 [0.5 points] What is the temperature T2 at the end of the second stage?
2.6 [0.5 points] Find the height H2. Express the reply in terms of H and alpha. Find the numerical value of H2.
2.7 [2 points] Find the frequency of small oscillations omega around the equilibrium position of the piston, assuming that the process is quasi-static and adiabatic. Express the reply in terms of g, H, gamma, and alpha. Find the numerical value of omega.

At the end of the second stage, a big number of small holes are made in the bottom

of the vessel, with the total area of holes S0=5.00 10-4. The size of each hole is much smaller than the mean free path of molecules. After some time the piston starts to move with a constant velocity u.
It is known that the average number of molecules N hitting the unit surface area per unit time is equal to

(1)

where nu is the so called mean thermal velocity of the molecules, R is the universal gas constant. The average kinetic energy of translational motion of molecules falling into the holes is

(2)

where kB is the Boltzmann constant.

Considering that the flow of heat through the walls and piston is negligible, answer the following questions:

2.8 [1 point] The final air pressure under the piston is of the form p3=A f(alpha), where A is a constant that depends on p0, and f(alpha) is a function of alpha. Find A and f(alpha). Find the numerical value of p3.
2.9 [2 points] The final velocity of the piston is given by u = B g (alpha), where B is a constant depending on d, S, R, T0, and mu, while g(alpha) is a function of alpha. Find B and g(alpha). Find the numerical value of u.
2.10 [1 point] The final temperature of the gas under the piston is of the form T3=C h(alpha), where C is a constant that depends on T0, and h(alpha) is a function of alpha. Find C and h(alpha). Find the numerical value of T3.

Problem 3
Nuclear droplet (10 points)

In this task, we consider the main characteristics and conditions for the stability of atomic nuclei. Let the atomic nucleus contains A nucleons (A is atomic weight of elements), namely, Z protons (Z is element's number in the table of chemical elements) and N=A-Z neutrons. The expression for the total energy of the nucleus can be written as

(1)

where M is the mass of nucleus, mp is the mass of the free proton, mn is the mass of the free neutron, c is the speed of light, and Ep is the potential energy of the nucleons in the nucleus.
The potential energy of nucleon-nucleon interaction can be described by the following semi-empirical formula by Weizsäcker

(2)

where MeV MeV MeV MeV.

Weizsacker semiempirical formula corresponds to one of the simplest model of the atomic nucleus, the so-called spherical liquid drop model, which rely on the analogy between the nucleus and drop an ordinary liquid. The mass and charge of the nucleus assumed to be uniformly distributed inside a sphere of some radius, and the nucleon fluid is characterized by some parameter sigma, which is an analogue of the surface tension of the liquid.

The formula for potential energy Ep takes into account the following contributions:

• surface energy, which takes into account the surface tension of nuclear matter in the liquid drop model;
• the energy of the Coulomb repulsion of the protons within the nucleus;
• the exchange interaction energy, reflecting the trend towards the stability of nuclei at N=Z;
• direct dependence on the number of nucleons A due to nuclear forces.

Also in the derivation of this semiempirical formula Weizsäcker used experimentally established the following dependence of the radius of the atomic nucleus of the number of nucleons

(3)

where R0 is a constant.
Based on all the above, give answers to the following questions:

3.1 [2 points] Find the electrostatic energy EC of a sphere of radius R, uniformly charged with total charge Q. Express answer in terms of charge Q, the dielectric constant epsilon0, and radius R of the ball.
3.2 [1 point] Find the numerical value of the coefficient R0 in the formula (3).
3.3 [1 point] Find the numerical value of the density rho_m of nuclear matter.
3.4 [1 point] Find the numerical value of the surface tension sigma of liquid nucleons.

Suppose now that the nucleus is broken into two parts with atomic weights kA and (1-k)A respectively, where 0<k<1. We can assume that the nuclear charge and the number of neutrons are distributed between the fragments as the atomic weight.
3.5 [2 points] Nuclear fission becomes energetically favorable under the condition Z2/A > f(k). Find an expression for the function f(k) and plot it schematically.
3.6 [0.5 points] Wit the accuracy up to two significant figures, find the limiting value (Z2/A)0 at which the spontaneous fission is theoretically possible.
Under the condition of § 3.5, the nucleus can stay for a long time. For example, the half-life of Uranium-235 nucleus is equal to 713 million years. Consequently, an instantaneous fission is prevented by some energy barrier, which disappears at some critical value. In fact, the nucleus will be broken when a significant deviation in its shape from spherical occurs.

For simplicity, we assume that a spherical nucleus undergoes such deformations under which the surface is the surface of a prolate ellipsoid of rotation, which in Cartesian coordinates by the equation

(4)

where a is small, and b is big semi-axes of the ellipsoid, respectively.

The volume of a prolate spheroid is given by

(5)

and its surface area can be calculated by the formula

(6)

Let a spherical nucleus undergoes such a deformation that b=R(1+epsilon) and a=R(1-lambda), so that epsilon, lambda <<1, and R is initial radius of the nuclear droplet.

3.7 [0.5 points] Find the relation between epsilon and lambda.
Calculations show that the energy of the electrostatic interaction of protons of the deformed nucleus is about Ecdeformed = .
3.8 [2 points] Find the expression and the numerical value of (Z2/A)_critical.

Known physical constants:
Elementary charge
The dielectric constant
The mass of the nucleon (proton or neutron) k
1eV in J

In addressing these tasks, you can use the formulas: