XIV International Zhautykov Olympiad in Mathematics
Almaty, 2018

January 12, 9.00-13.30
First day
(Each problem is worth 7 points)

1. Let α, β, γ be the angles of a triangle opposite to the sides a, b, c respectively. Prove the inequality

$$2\left(\cos^2\alpha+\cos^2\beta+\cos^2\gamma\right) \geq \frac{a^2}{b^2+c^2}+\frac{b^2}{a^2+c^2}+\frac{c^2}{a^2+b^2}.$$

2. Points N, K, L lie on the sides AB, BC, CA of a triangle ABC respectively so that $AL = BK$ and CN is the bisector of the angle C. The segments AK and BL meet at the point P. Let I and J be the incentres of the triangles APL and BPK respectively. The lines CN and IJ meet at point Q. Prove that $IP = JQ$.

3. Prove that there exist infinitely many pairs (m, n) of positive integers such that $m+n$ divides $(m!)^n + (n!)^m + 1$.