THEORETICAL COMPETITION

January 15, 2016

Please read this first:

1. The time available for the theoretical competition is 4 hours. There are three questions.
2. Use only the pen provided.
3. You can use your own calculator for numerical calculations. If you don’t have one, please ask for it from Olympiad organizers.
4. You are provided with ***Writing sheet*** and additional paper. You can use the additional paper for drafts of your solutions but these papers will not be checked. Your final solutions which will be evaluated should be on the ***Writing sheets***. Please use as little text as possible. You should mostly use equations, numbers, figures and plots.
5. Use only the front side of ***Writing sheets.*** Write only inside the boxed area.
6. Begin each question on a separate sheet of paper.
7. Fill the boxes at the top of each sheet of paper with your country (**Country),** your student code (**Student Code**), the question number (**Question Number)**, the progressive number of each sheet **(Page Number**), and the total number of ***Writing sheets*** used (**Total Number of Pages**). If you use some blank ***Writing sheets*** for notes that you do not wish to be evaluated, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, arrange all sheets for each problem in the following order:
   - Used ***Writing sheets*** in order;
   - The sheets you do not wish to be evaluated
   - Unused sheets and the printed question.

   Place the papers inside the envelope and leave everything on your desk. You are not allowed to take any paper out of the room.

Problem 1 (10.0 points)

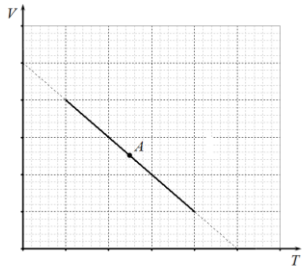
This problem consists of three independent parts.

Problem 1A (4.0 points)

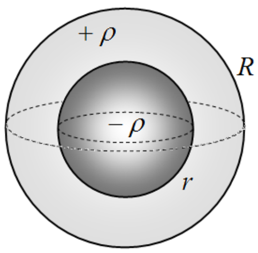
A homogeneous planet of radius R has no atmosphere and does not rotate. A stone is thrown from the surface of the planet at an angle α to the horizontal with the speed v_0, which is equal to the orbital velocity for the surface of the planet. Find the maximum height of ascent of the stone above the surface of the planet. At what range from its initial point, measured along the surface of the planet, the stone will hit the surface?

Problem 1B (3.0 points)

One mole of an ideal monatomic gas performs a process whose chart in VT coordinates completely lies on a straight line. Find the heat capacity of the gas at the point A, equidistant from the points of intersection of the process line with the coordinate axes.



Problem 1C (3.0 points)



Two spheres with the radii r and R ($r < R$) \with the common center divide the space into three domains. The interior of the small sphere is uniformly charged with the volume charge density $-\rho$, the domain in between the spheres is uniformly charged with the volume charge density $+\rho$, and there is no charge outside the larger sphere. Find the ratio of the radii R/r, at which the potential in the center of the symmetry of the system is equal to the potential at infinity.

Problem 2. Equilibrium in terms of potential energy (10.0 points)

One of the widely known principles of the general physics is that every system tends to decrease its potential energy, and the stable equilibrium position corresponds to the state with the minimum of its value.

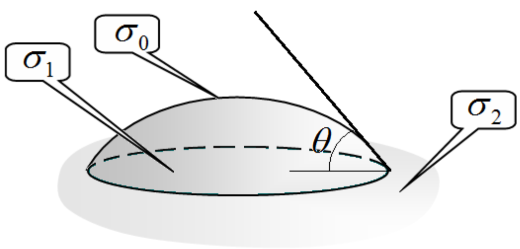
In this problem the interaction of the liquid with the solid surface is studied. To describe this interaction the following parameters are introduced:

σ_0 is the surface tension at the interface between the liquid and the air;

σ_1 is the surface tension at the interface between the liquid and solid;

σ_2 is the surface tension at the interface between the solid and the air;

θ is the contact angle (wetting angle).



The values σ_0, σ_1, σ_2 designate the surface energy per unit area of contact between media.

In all parts of the problem, use the following numerical values for water:

the surface tension $\sigma_0 = 0,072 \frac{N}{m}$;

the contact angle $\theta = 20°$;

the mass density $\rho = 1,0 \cdot 10^3 \frac{kg}{m^3}$;

the acceleration of gravity $g = 9,8 \frac{m}{s^2}$.

1. Introduction (1.0 points)

1. **[1.0 points]** Prove that the change in the surface energy at the liquid-solid interface is found as

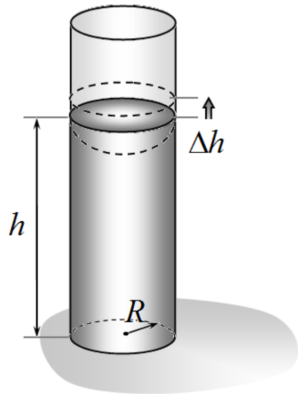
$$\Delta U_s = -\sigma_0 \cos\theta \; \Delta S, \tag{1}$$

where ΔS stands for the change in the area of the contact between the liquid and solid.

2. Water in a vertical cylindrical tube (2.0 points)

The open tube with the inner radius $R = 1{,}0mm$ is lowered vertically so that its lower end touches the surface of the water.

Let a water level in the tube be at a certain height h, which does not necessarily correspond to its equilibrium value.



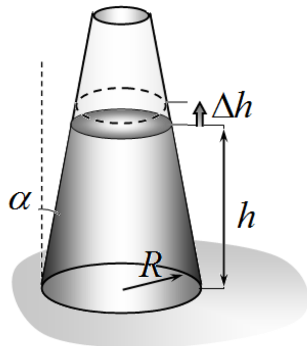
2.1 **[0.5 points]** Find the formula for the change in the surface energy of the system ΔU_S that corresponds to an additional small rise of water level Δh in the tube.
2.2 **[0.5 points]** Find the formula for the change in the potential energy ΔU_G of the liquid in the gravitational field that corresponds to an additional small rise of water level Δh in the tube.
2.3 **[1.0 points]** Using the principle of the minimum potential energy, find the formula for the height of the water in the tube h_0 in equilibrium position. Calculate its numerical value from the quantities provided above.

3. Water in a vertical conical tube (4.0 points)

A long conical tube is vertically lowered into the water so that its lower end touches the surface of the water. The inner radius of the tube at its lower base is equal to $R = 1{,}0mm$, and its inner radius at the upper base is close to zero. The tube walls make an angle α with the vertical.

Note: in the following neglect any change in the surface energy at the interface between the liquid and the air.

Let a water level in the tube be at a certain height h, which does not necessarily correspond to its equilibrium value.



3.1 **[0.5 points]** Find the formula for the change in the surface energy of the system ΔU_S that corresponds to an additional small rise of water level Δh in the tube.
3.2 **[0.5 points]** Find the formula for the change in the potential energy ΔU_G of the liquid in the gravitational field that corresponds to an additional small rise of water level Δh in the tube.
3.3 **[1.0 points]** Find the equation that determines the height of water in the tube in equilibrium position and rewrite it in terms of σ_0, θ, α and the value of h_0 found in 2.3.
3.4 **[1.0 points]** Let an angle be $\alpha = 1{,}0 \times 10^{-2}\ rad$. The tube is partially filled with water up to a certain level H. Find the dependence of the ultimate height of the water level in the tube as a function of H.
3.5 **[1.0 points]** Specify the range of angles α (providing its numerical values) at which the water completely fills the tube.

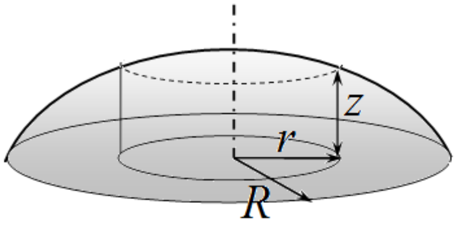
4. Outflow of water (3.0 points)

A bottle is completely filled with water, sealed tightly with the cork and turned upside down. Two identical round holes of radii R are drilled in the cork.

4.1 **[3.0 points]** At what minimum value of the hole radius the water will pour out of the bottle?

Mathematical tips

Rather small convex spherically shaped surface can be approximately described by a function



$$z = h\left(1 - \frac{r^2}{R^2}\right),$$

where R denotes the radius of the bulge with h being its height such that $h << R$.

Then, up to higher orders the area of the spherical part of the bulge is found as

$$S = \pi\left(R^2 + h^2\right),$$

and its potential energy in the gravitational field is derived as

$$U = \frac{\pi R^2 h^2}{6}\rho g .$$

Problem 3. Nonlinear capacitor (10.0 points)

The electrical circuit contains the voltage source U_0 \connected in series with the resistor $R = 1{,}00\ k\Omega$ and the nonlinear capacitor, whose capacitance depends on the voltage across it and is graphically shown in the figure below.

Note: *to solve this problem you may need to use some piece of squared paper provided under the graph,*

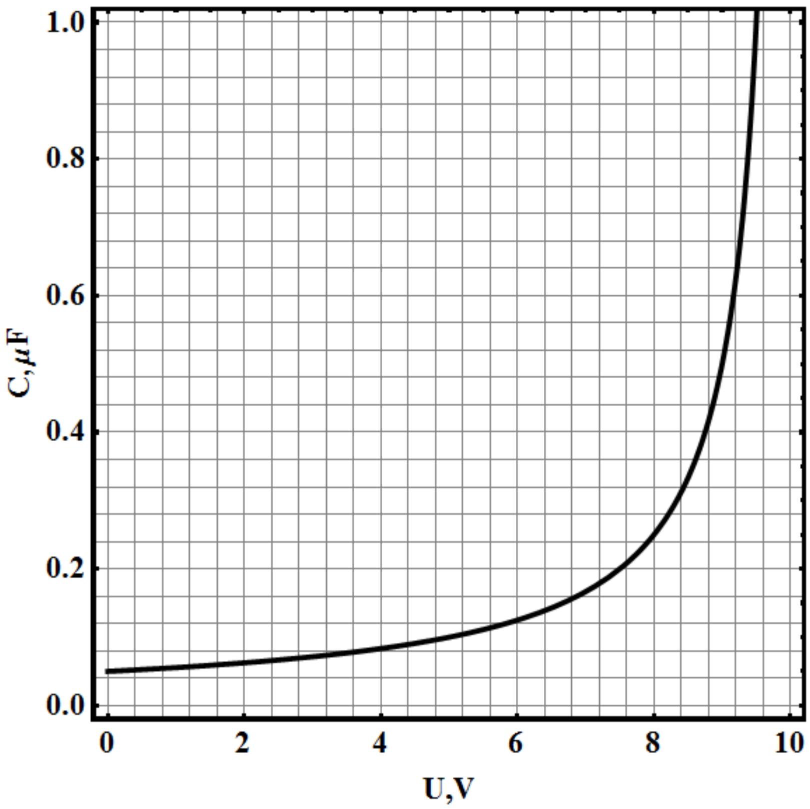
1. **[0.75 points]** Assume that $U_0 = 5{,}0\ V$. Find the charge of the capacitor, which appears on its plates after a sufficiently long period of time.

Suppose that at the initial time moment the charge of the capacitor is zero, and the voltage source provides $U_0 = 10$ V. It is seen from the graph that the capacitance of the capacitor at this voltage goes to infinity, i.e. $C(10\ V) = \infty$.

2. **[0.25 points]** How long does it take for a capacitor to be charged to the voltage $U_0 = 10$ V?
3. **[3.0 points]** Find the time moment t, when the charge of the capacitor is equal to $q = 4{,}0\ \mu Cl$.
4. **[0.5 points]** Find the time interval Δt at which the charge of the capacitor grows from $q_0 = 4{,}0\ \mu Cl$ to $q = 8{,}0\ \mu Cl$.
5. **[0.5 points]** Find the charge of the capacitor at the time moment $t_0 = 3{,}0\ ms$.

Let a source provide a constant voltage with a small portion of alternating voltage such that $U_0 = U + \delta U \sin \omega t = [5{,}000 + 0{,}100 \sin \omega t\,]$ V, where $\omega = 2500\ rad/s$. After a sufficiently long period of time electrical oscillations of voltage and current are set up in the circuit.

6. **[0.5 points]** What is the phase difference φ between the voltage oscillations across the capacitor and the resistor?
7. **[4.0 points]** Find the dependence of the electric current in the circuit $I(t)$ as a function of time.
8. **[0.5 points]** Find the voltage across the capacitor $U_C(t)$ as a function of time.


1.0
0.8
0.6
0.4
0.2
0.0
C, μF
0
2
4
6
8
10
U,V