THEORETICAL COMPETITION

January 13, 2015

Please read this first:

1. The time available for the theoretical competition is 4 hours. There are three Problems.
2. Use only the pen provided.
3. You can use your own calculator for numerical calculations. If you don't have one, please ask for it from Olympiad organizers.
4. You are provided with ***Writing sheet*** and additional paper. You can use the additional paper for drafts of your solutions but these papers will not be checked. Your final solutions which will be evaluated should be on the ***Writing sheets***. Please use as little text as possible. You should mostly use equations, numbers, figures and plots.
5. Use only the front side of ***Writing sheets.*** Write only inside the boxed area.
6. Begin each Problem solution on a separate sheet of paper.
7. Fill the boxes at the top of each sheet of paper with your country (**Country),** your student code (**Student Code**), the question number (**Problem Number)**, the progressive number of each sheet **(Page Number**), and the total number of ***Writing sheets*** used (**Total Number of Pages**). If you use some blank ***Writing sheets*** for notes that you do not wish to be evaluated, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, arrange all sheets for each problem in the following order:
    - Used ***Writing sheets*** in order;
    - The sheets you do not wish to be evaluated
    - Unused sheets and the printed question.

    Place the papers inside the envelope and leave everything on your desk. You are not allowed to take any paper out of the room.

Problem 1. Plotting (7 points)

For each part of this Problem, please, use the provided separate sheets of papers with figures in which you should do all necessary plotting. Plotting and its justification should be made on the same separate sheets.

Problem 1.1 (2.0 points)

Three small positively charged balls (whose charges are different) with masses $m, 2m, 3m$, are connected to each other by inextensible nonconductive threads such that the balls are located at the vertices of the equilateral triangle $A_1A_2A_3$ (see figure 1.1). When the threads connecting balls have been cut down (not simultaneously) all balls start to move on the same plane. In figure 1.1 the positions B_1 and B_2 of two balls are shown at some time moment. With the help of geometric constructions, indicate position B_3 of third ball at the same time moment.

Problem 1.2 (2.0 points)

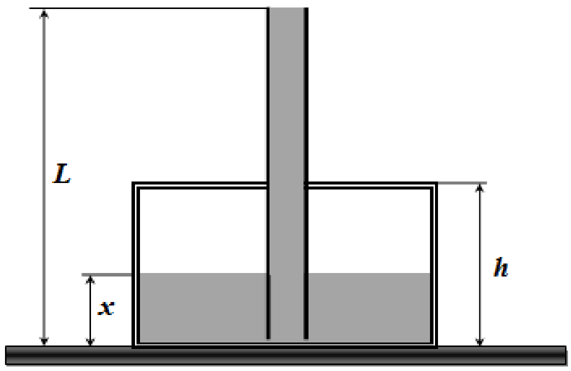
In figure 1.2, logarithmic scale is used to show straight lines of two processes performed by an ideal diatomic gas (P_0, V_0 are some constants). Plot a cyclic process lying in between those lines, which has maximum efficiency. In the same figure, extreme points of that cycle are indicated, with A_1 being the point of the minimum volume and A_3 being the point of the maximum volume of the gas. Find the efficiency of the cycle.

Problem 1.3 (3.0 points)

In figure 1.3, positions are shown of the point light source S and its image S' created by a thin lens. In the same figure, OO_1 is the main optical axis of the lens. Plot an image of the point source S_1 created by the same thin lens.

Problem 2. Vessel with water (7 points)

In the cylindrical vessel with the cross section area $S = 0.500\ m^2$ and height $h = 0.500\ m$ a tube of length $L = 2.00\ m$ with opened ends is inserted vertically through the hermetically sealed lid. The lower end of the tube is a bit above the bottom of the vessel. Water with the density $\rho = 1000\ kg/m^3$ is poured into the vessel as shown in the figure on the right. The cross section area of the tube is much smaller than the cross section area of the vessel and the vessel wall material conducts heat very well. Assume that the atmospheric pressure is $p_0 = 1.01 \cdot 10^5\ Pa$, the ambient temperature is $T_0 = 293\ K$ and the acceleration of gravity is $g = 9.80\ m/s^2$.



1. **[2.0 points]** Find the height of the water level in the vessel $x = x_0$ at the time moment when the tube is completely filled with water. Express your answer in terms of p_0, ρ, g, h, L, and find its numerical value.

Now, assume that the walls of the vessel and the tube are coated with a material which does not conduct heat at all. The air inside the vessel is then heated fast enough such that the water does not have enough time to warm up.

2. **[0.5 points]** Find air pressure inside the vessel $p(x)$ as a function of x. Express your answer in terms of p_0, ρ, g, L, x.
3. **[1.0 points]** Find air temperature inside the vessel $T(x)$ as a function of x. Express your answer in terms of p_0, ρ, g, L, x.
4. **[1.0 points]** Find the temperature T_m to which the air in the vessel must be heated in order to displace all the water from the vessel. Express your answer in terms of p_0, ρ, g, L, T_0 and find its numerical value.

5. **[2.5 points]** Find the amount of heat Q, which must be given to the air in the vessel in order to displace all the water from the vessel. Express your answer in terms of $p_0, \rho, g, h, L, S,$ and find its numerical value.

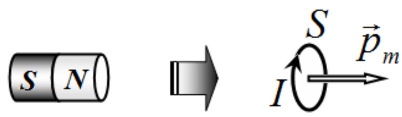
Problem 3. Delay and attenuation (16 points)

In this problem, do not take into account the finiteness of the propagation speed of the electromagnetic interaction.

Part 1: Magnet

1.1 Theoretical introduction

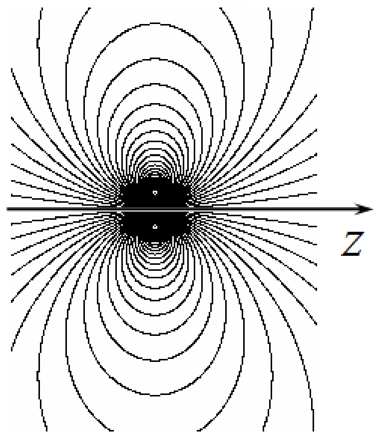
The magnetic field generated by a uniformly magnetized ferromagnetic cylinder (permanent magnet) is equivalent, at very large distances, to the field produced by a circular coil with a constant electric current.



The cylindrical magnet, as well as the coil with the current, are characterized by magnetic moment p_m, which is defined for the current loop as the product of the current and the area of the loop,

$$p_m = IS\ .$$

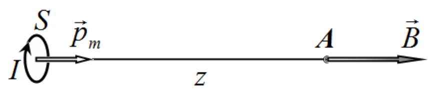
Such a source of magnetic field is also referred to as a *magnetic dipole*. The figure shows the magnetic field lines of such a dipole.



1.1.1. **[0.75 points]** Show that the magnetic field, B_z, on the axis of the dipole is determined at large distances by the formula

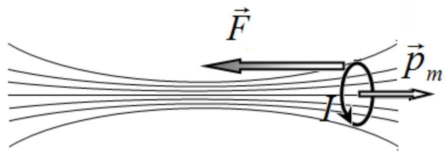
$$B_z = b\frac{p_m}{z^\beta},$$

where z is the coordinate measured along the axis of the dipole from its center. Find the values of the parameters b and β in the above formula.



1.1.2. **[1 point]** Let the coil with a current (i.e., magnetic dipole) with the magnetic moment p_m is influenced by an inhomogeneous axially symmetric field, the induction of which along the z-axis depends on z as function $B_z(z)$. Dipole axis coincides with the axis of symmetry of the field. Show that the force acting on the dipole from the magnetic field is given by

$$F_z = -p_m\frac{dB_z}{dz}.$$



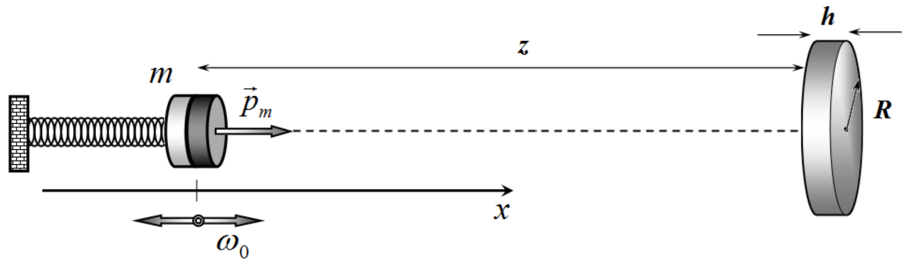
1.2 Oscillations of the magnet

Cylindrical magnet of mass m and magnetic moment p_m, is attached to the spring of stiffness k such that it can oscillate along the horizontal axis which is directed along the magnetic moment.

1.2.1. **[0.25 points]** Find the frequency ω_0 of free oscillations of the magnet in the absence of external fields.

At some distance z from the equilibrium position of the magnet a small metal disc is placed such that its axis coincides with the axis of the magnet. The disk has radius R and thickness h ($h \ll R \ll z$), the electrical resistivity of the disk material is ρ, and the magnetic permeability is put equal to $\mu = 1$. The magnet is moved from the equilibrium position and starts performing small oscillations described by function $x(t)$, where $x \ll z$.

1.2.2. **[2 points]** Find the force $F(x, v)$ exerted by the disc on the magnet as a function of its coordinate x and velocity v. Write down the equation of motion of the magnet.



1.2.3. **[0.75 points]** Find the relative change $\Delta\omega/\omega_0$ in the oscillation frequency of the magnet caused by the influence of the disc.
1.2.4. **[0.25 points]** Assuming that the attenuation is rather weak, obtain the characteristic attenuation time of the oscillations of the ball.
1.2.5. **[1.5 points]** Show that the loss of the mechanical energy of the magnet is equal to the amount of heat released in the disk for the same time period.

Mathematical tip

The equation of attenuating oscillations

$$d^2x/dt^2 + 2\beta dx/dt + \omega_0^2 x = 0$$

has the solution

$$x(t) = A\exp\left(-\frac{t}{\tau}\right)\cos(\omega t + \varphi),$$

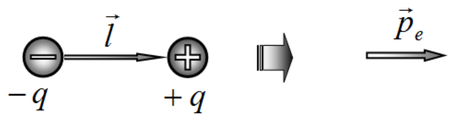
where $\omega = \sqrt{\omega_0^2 - \beta^2}$ is the frequency of attenuating oscillations, $\tau = 1/\beta$ is the characteristic attenuation time, and the parameters A, φ are determined by the initial conditions.
It is known that if $x \ll 1$ the following approximate inequality holds $(1+x)^\alpha \approx 1 + \alpha x$.

Part 2: Electric

2.1 Theoretical introduction

The system of two identical in magnitude and opposite in sign charges $(-q, +q)$, located at some fixed distance l from each other is called an *electric dipole* and is characterized by the dipole moment

$$p_e = ql.$$



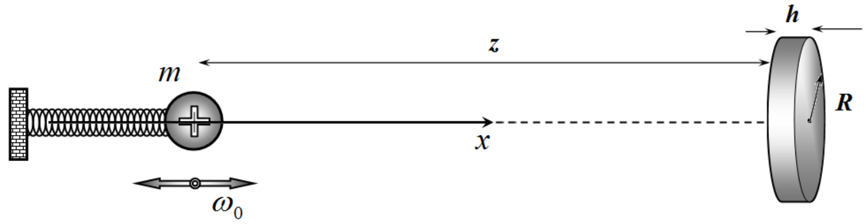
2.1.1. **[0.75 points]** The electric field generated by the dipole on its axis at the distance $z \gg l$, is defined by the formula

$$E = a\frac{p_e}{z^\alpha}.$$

Obtain the parameters a, α in this formula.

2.2 Fluctuations of charged ball

A small ball of mass m carrying the electric charge q is attached to a non-conductive spring of stiffness k and can oscillate along the horizontal axis x. At some distance z from the position of the equilibrium of the ball, a small metal perfectly conducting disk is fixed such that its axis coincides with the x axis. The disc has radius R and thickness h ($h \ll R \ll z$).



2.2.1. **[0.75 points]** Find the shift of the equilibrium position of the ball caused by the influence of the disc.
2.2.2. **[0.75 points]** Find the relative change in the oscillation frequency of the ball, $\Delta\omega/\omega_0$, caused by the influence of the disc.

Assume now that the electrical resistivity of the material of the disk is ρ (not zero).

2.2.3**. [1.5 points]** Obtain an equation describing the time variation of the induced dipole moment of the disc (i.e., the equation relating the dipole moment of the disk p and its rate of change over time dp/dt).

2.2.4. **[0.25 points]** Assuming that the disk is a capacitor whose plates are connected to each other by a resistor, obtain the characteristic time of the equivalent RC-circuit. Express your answer in terms of resistivity ρ of the material of the disk.

Assume in the following that the characteristic time obtained in 2.2.4 is much smaller than the oscillation period of the ball.

2.2.5. **[0.25 points]** Write down the relation between ω and ρ expressing the above stated assumption.

In the case of ideal conductivity of the disk the oscillations of the ball do not attenuate. At low resistivity of the material of the disc the attenuation of oscillations should also be small, and such oscillation can be approximately regarded as harmonic ones.

2.2.6. **[2 points]** Using this approximation and the equation derived in 2.2.3, obtain the expression of the dipole moment p of the disk via coordinate x and the speed v of the ball.

2.2.7. **[1.5 points]** Find expression for the force exerted on the ball by the disk. Write down the equation of motion of the ball.

2.2.8. **[0.25 points]** Find the characteristic attenuation time of the oscillations of the ball.

2.2.9. **[1.5 points]** Show that the loss of mechanical energy of the ball is equal to the heat generated in the disk for the same time period.